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Experiments on ring wave packet generated by water drop

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The propagation of ring wave packet (composed of carrier waves modulated by an envelope) generated by a water drop was studied experimentally. It is a localized wave packet propagating with constant velocity and low diffusion. The wavelength, the amplitude and the waveform of the carrier waves, the velocity of the carrier waves and the packet have been measured. The measured wavelength λ_e of the carrier waves and the measured group velocity C_{ge} of the packet are near the minimum point of the dispersion curve of group velocity, which may be the main reason why the packet can propagate with low diffusion.

water waves, group velocity, phase velocity, ring waves, water drop, waveform measurement, solitary waves

In physics, the concept of wave motion originates from the observation of water surface waves. The ring waves generated by water drops are well known phenomena in nature. In optics, the concept of subwave, which is like the ring wave geometrically in 2-dimension case, is important in the formation of wave theory of optics. When Le Mehaute[1] attempted to establish the theory of water waves created by the impact of small objects in 1988, he could find only one related photograph of the rings caused by raindrops taken by Crapper[2]. For the explanation of the photograph, Crapper introduced a wavelength λ_m, where the group velocity reaches to a minimum. The wavelengths of the inner rings are nearly equal to λ_m, while those of the outer rings are shorter than λ_m. According to Le Mehaute's study, the number of rings increases with time and distance. The ring waves generated by raindrops can impact the detecting of radar[3]. An optical method was developed to measure wave profiles along the radius[3]. They concluded the characteristic wave number is 0.2 mm^{-1}.

In the spring of 2005, we observed a localized ring wave packet in a water lily pond in our campus. We found that such a wave packet appeared accidentally when a duck dipped the water with its mouth. This observation stimulated our original idea, and excited us to carry out the experiments to quantitatively study this kind of wave packet. In this paper, the propagation process of the ring wave packet excited by a water drop was recorded by a digital video camera with a speed of 30 fps. The packet is composed of carrier waves modulated by an envelope. The wavelength λ_e of the carrier waves, the group velocity C_{ge} of the packet and the phase velocity C_{pe} of the carrier waves were measured. From the dispersion curve (Figure 5) of the group velocity C_g against the wavelength λ, one can find that λ_e is nearly equal to λ_m, and C_{ge} is nearly equal to the minimum value of the group velocity. This means that the dispersion of group velocity is quite small, but the phase velocity is still located in the linear dispersion range of the dispersion curve. Figure 4 shows the propagation of the carrier relative to the envelope. The wave packet can keep its waveform with low diffusion by propagating more than 1.4 m at a constant velocity in more than 10 s. The low diffusion propagation maybe caused by the fact

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that the value of λ_e is very close to λ_m.

From the viewpoint of dynamics, the wave that we study is a kind of gravity-capillary waves. The gravity-capillary water waves of solitary type have been studied theoretically[4] and experimentally[5]. The observed V-shaped depression curve in ref. [5] agrees well with the theoretical curve in ref. [4]. The waves propagated as free solitary waves in 1.1 s and were finally damped by viscosity[5].

1 Experimental method

The experiments were carried out near the zigzag bridge in Yuan-Ming Yuan Imperial Garden, where the water surface was under the shadow of a building, so that the sunlight is hardly interfered with the photography and there were few disturbances by wind and other waves. The ring waves were generated by periodically ejecting water drops with a period of τ = 1.6 s. The water drops were produced by a siphon with an outlet diameter of 2 mm, each drop weighing 39 mg, which was selected in an observation of waves in a basin[6]. The drops were ejected from a height of h_1=147 cm above the water surface, which was the distance between the platform of bridge and water surface. The propagation of waves was recorded by a digital video camera with resolution 640×480 pixels at 30 fps. The camera was installed h_2=245 cm above the water surface and was above the outlet of the water drops. The optical axis of camera deviated an angle from the normal of water surface. Video 1 and Video 2 are experimental records on different dates, and Figures 1—4 are cut from Video 1. Figures 2—4 are locally enlarged.

2 Results

2.1 The velocity of wave packets

From Figure 1, one can find there are six groups of ring stripes, each group corresponding to a wave packet generated by a water drop, and the radius of the outmost packet is 1.4 m. The bright and dark stripes are caused by the scattering of the light by the waved surface, and the bright stripes or dark ones are related to the position where the normal of the water surface inclined to or lapsed from the camera. In the lower left area of Figure 1, a white round object is used as a reference for length measurements. Its real diameter is D = 6.55 cm, but in the photograph it is d = 0.74 cm, and their ratio is r = D/d = 8.94. The distances between the adjoining rings are marked as d_1, d_2, d_3 and d_4. The original data of the distances, listed in the left of Table 1, were obtained by measuring them on Figure 1 by the scale of the reference object. Considering that the optical axis of the camera deviated an angle from the normal of the water surface such that the objects in the photograph were distorted, the results should be corrected. The camera is fixed vertically above the position B with height h_2 and the axis of it points at the center O of the photograph. The distance L between B and O can be measured by the scale of the reference object. The angle between the water surface and the imaging surface of the camera can be calculated by the values of h_2 and L, and then the values of d_1—d_4 can be corrected by geometrical method. The corrected values are listed in the right part of Table 1. For water drops with a period of τ =1.6 s, the corresponding

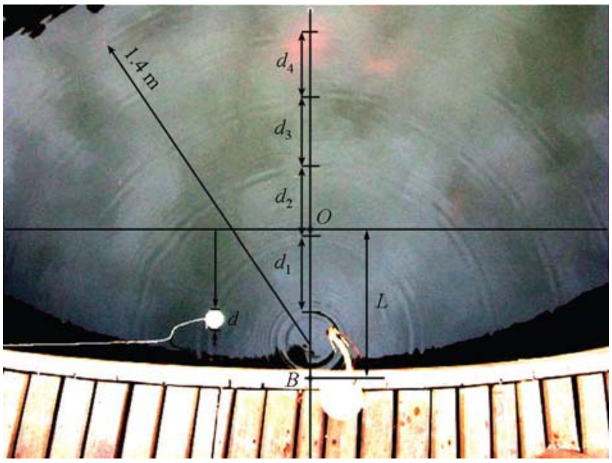

Figure 1 The photograph of ring waves generated by water drops. There are six groups of ring stripes in the figure, and the radius of the outmost ring is 1.4 m. O denotes the center of the photograph and B is the position above which the camera is fixed vertically. The real diameter of the white reference object on the left of the ring center is D=6.54 cm, and that in the photograph is d=0.74 cm, the ratio r=D/d=8.84. d_1, d_2, d_3 and d_4 denote the distances between the adjoining rings.

Table 1 The measurement of the velocity of the wave packet

	Original data a)		Corrected b)	
	Distance (cm)	Velocity (cm/s)	Distance (cm)	Velocity (cm/s)
d_1	24.3	15.2	24.9	15.6
d_2	23.1	14.4	24.6	15.4
d_3	22.8	14.3	25.2	15.8
d_4	21.5	13.4	24.7	15.4

a) Evaluated by comparing with the reference object. b) Considering that the optical axis of the camera deviated an angle from the normal of the water surface such that the objects in the photograph have been distorted, the results should be corrected. Data are corrected according to the parameters h_2=245 cm and α=11.4°, where h_2 denotes the height of the camera and α the angle between the optical axis of the camera and the normal of the water surface.

velocities are evaluated and also listed in Table 1. The average group velocity is C_{ge}=(15.5±0.3) cm/s.

2.2 The velocity of carrier waves

Two locally enlarged pictures are shown in Figure 2. One can find the carrier waves in the packet and one bright stripe of carrier waves transports towards to the reference object. Figure 2(a) and (b) are two subsequent pictures with a time interval of 0.10 s. We marked and measured the distance between the center of the bright stripe and one edge of the reference, and thus obtained the velocity of the carrier waves C_{pe}=24.8 cm/s.

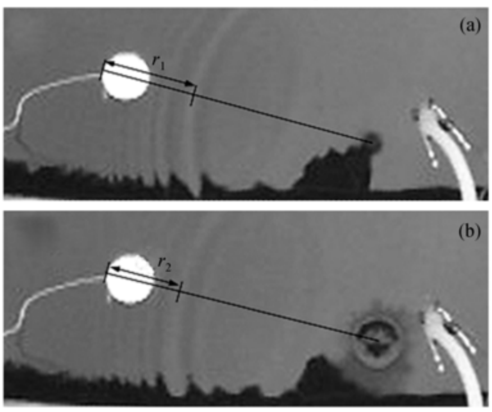

Figure 2 The measurement of the velocity of the carrier waves. The time interval of the two photographs is 0.10 s. The distances between the center of the bright stripe and the edge of the reference are r_1=12.84 cm and r_2=10.36 cm, respectively. r_1–r_2=(2.48±0.18) cm, and the velocity of the carrier wave is (24.8±1.8) cm/s.

2.3 The wavelength, amplitude of the carrier waves and nonlinearity

From the left part of Figure 3, we can obtain the wavelength of the carrier waves, i.e. λ_e=3.3 cm. In the right part, at the point 31 cm away from the point of the water drops impacting, the image of the edge of the bridge in water is distorted by the wavy surface. The displacement of the image along the normal of the edge of the bridge is measured as A_i =1.5 cm. The displacement of the image of the bridge in water along the normal of the wave front is $\Delta=2h_1\varphi$, where φ is the slope angle of the water surface caused by the wave and h_1 is the distance between the platform of the bridge and the water surface. Recorded by the camera, the displacement projecting on the water surface is $\Delta_1=\Delta h_2/(h_1+h_2)$, where h_2 is height of the camera from the water surface, so the displacement of the image of the edge the bridge in water is proportional to the slope of the water surface. Assuming the wave is locally a sine wave, we have $\varphi = kA$, where A is the amplitude of the wave, and $k=2\pi/\lambda$. Figure 3 also shows the angle between the edge of the bridge and the normal of the wave front, namely, θ=18°. And we have $A_i=\Delta_1\sin\theta$, so the amplitude can be written as

$$A=\lambda_e A_i(h_1+h_2)/(4\pi h_1 h_2\sin\theta). \quad (1)$$

The evaluated value is A = 0.14 mm by substituting the related values into eq. (1).

In Navier-Stokes equations, the ratio of the linear and the nonlinear term is estimated to be $\lambda/2\pi A$. In the present experiment, $\lambda_e/2\pi A$=38, and in Russell's experiment[7], the ratio of amplitude to wavelength is 1/30, $\lambda/2\pi A$=5. The nonlinearity in our experiment is about 1/8 of that in Russell's experiment, but it still has sufficient effects. The following discussion shows that the nonlinearity appears only about 1—2 s after water drop impacts on water surface.

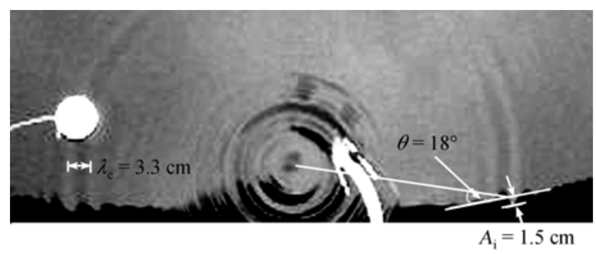

Figure 3 The measurement of the wavelength and the amplitude. The left part of the figure is used to measure the wavelength. The distance between the adjoining crests is the wavelength, i.e. 3.3 cm. The right part is used to measure the amplitude. The displacement of the image of the bridge edge in water is A_i=1.5 cm. The point of measuring amplitude is 31 cm away from the point of the water drops impacting.

2.4 The velocity of the carrier waves relative to the envelope

The image of the edge of the bridge in water is distorted by the wavy surface and makes a waveform imaging on the picture (see the lower right part of Figure 1). According to the discussion in previous section, the crest and the trough are corresponding to the positions of the real wave where the slopes reach the maximum and the minimum. It looks like a carrier wave modulated by an envelope, shown in detail in Figure 4(a). In the previous paragraph, we obtained the velocities of the wave packet and the carrier waves, namely C_{ge}=15.5 cm/s and C_{pe}=24.8 cm/s respectively; thus the velocity of the carrier waves relative to the envelope is

$$c'_p = c_{pe} - c_{ge} = (24.8-15.5)\ \text{cm/s} = 9.3\ \text{cm/s}. \quad (2)$$

In Figure 4(a), the trough of the carrier waves V_1 is located on the trough of the envelope. After a short time of Δt, the next trough of the carrier waves V_2 replaces the position of V_1. Since λ_e=3.3 cm, we have

$$\Delta t = \frac{\lambda_e}{c'_p} = \frac{3.3}{9.3}\,\mathrm{s} = 0.35\,\mathrm{s}, \tag{3}$$

which corresponds to a time interval of 11—12 frames, for the frame rate of 30 fps. Local enlarged pictures of twelve sequent frames are listed in Figure 4(b). Two white lines are drawn cross the positions of V_1 and V_2 respectively. We can see that during the traveling to the right of the wave packet, the deepness of trough V_1 decreases and that of trough V_2 increases gradually. In the 12th picture trough V_2 has replaced trough V_1 and become the new trough of the envelope.

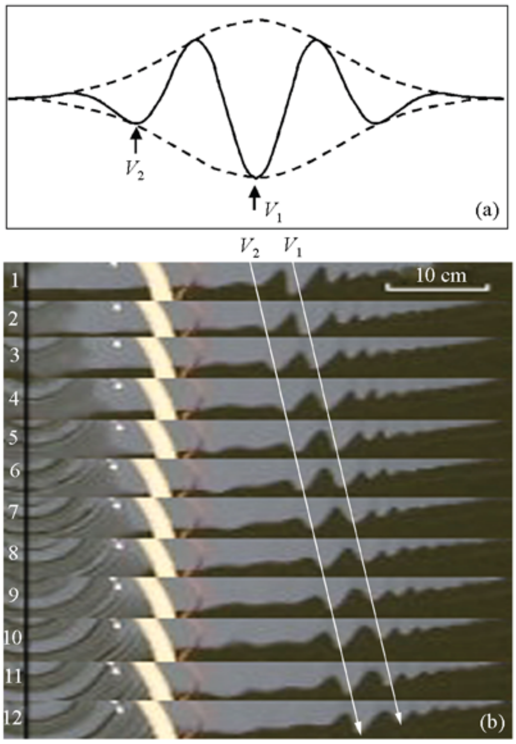

Figure 4 The motion of the carrier wave relative to the envelope. (a) Sketch of the motion of the carrier wave relative to the envelope. Four wavelengths of carrier wave are included in a Gauss envelope. (b) Real propagation of the carrier wave modulated by envelope, taken from Video 1, corresponding to the lower right part of Figure 1. The time interval is 1/30 s.

2.5 The values of phase velocity C_P and group velocity C_g on the dispersion curve

According to the linear wave theory[2], the phase velocity C_P and group velocity C_g can be calculated as functions of wavelength λ, namely

$$C_p = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi\sigma}{\rho\lambda}}, \tag{4}$$

$$C_g = C_p - \lambda\frac{dC_p}{d\lambda}, \tag{5}$$

where σ = 0.045 N/m denotes the coefficient of surface tension, g = 9.8 m/s^2 the gravity acceleration, ρ = 1000 kg/m^3 the density of water and λ the wavelength. The result is shown in Figure 5. The coefficient of surface tension σ is well selected in the calculation to lessen the differences between the measured values and the calculated values of both C_p and C_g at λ_e.

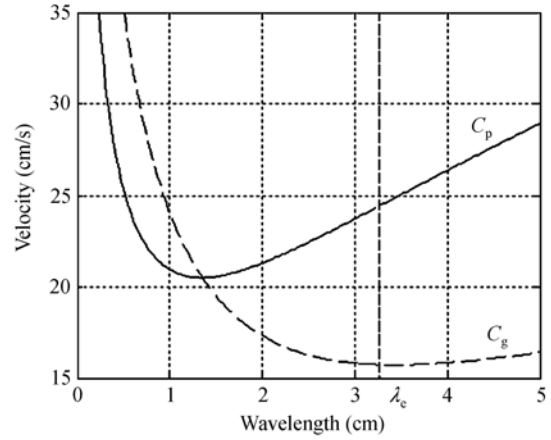

Figure 5 Dispersion curve of water waves. Solid line, phase velocity C_p; dashed line, group velocity C_g. λ_e is the measured wavelength of the carrier wave.

3 Discussion

3.1 The distance that the wave can propagate

Due to geometrical diminishing, the energy density of a ring wave is approximately proportional to $1/r$, i.e. the energy density at 1.5 m decreases to about one tenth of that at 15 cm. A similar experiment has been carried out in a basin with the water surface diameter being 68 cm. A similar carrier wave modulated by an envelope propagated to the wall of the basin and was reflected to the center of the basin again and again for more than 7 times and the total distance was more than 4.7 m. This experiment is still going on.

3.2 The concept of group velocity

The concept of group velocity in general physics is usually presented according to the original work of Rayleigh[8]. The superposition of two trains of sine waves with similar wavelengths and frequencies forms a carrier wave modulated by a wave with a low frequency. The envelope propagates with group velocity while the carrier wave propagates with phase velocity. In fact, it is difficult to observe this phenomenon directly in dispersive media. Figure 4 shows the propagation of a carrier wave relative to an envelope directly. Figure 4(a) can be approximately treated as the superposition of two trains of sine waves with

$$k_1=2\pi/\lambda_1 \text{ and } k_2=2\pi/\lambda_2,$$

where $k_1+k_2=\pi/\lambda_c$, $k_1-k_2=\pi/\lambda_{en}$, and $\lambda_c=\lambda_e$=3.3 cm, $\lambda_{en}=8\lambda_c$, denoting the wavelengths of the carrier wave and the envelope respectively. We can obtain λ_1 =2.93 cm, λ_2=3.77 cm. One can find that the range of the wavelength is located at the flat area of the dispersion curve of group velocity (Figure 5). In this range, the phase velocity is still dispersive, and the experiment can directly show the concepts of group velocity and phase velocity.

3.3 The formation of packet generated by water drop

By showing Video 1 frame by frame from the time of the water drop impacting on the water surface, one can find that the ring wave generated by the initial drop was much intensive at the beginning and then disperses gradually after the 9—12th frames. The carrier wave modulated by an envelope is formed after the 30th frame and propagates more than 1.4 m with little dispersion. In our experiments the carrier wave modulated by an envelope is generated by the water drop or column spattered by the initial water drop.

According to the existing reports, the shape splashed by a water drop is quite complicated[9]. Recently crowns[10] and bouncing drops[11] have caught much attention. Besides the crown (see Figure 6(a)), a tower-like water column with a drop on the spire (see Figure 6(b)) was recorded in our experiment. The formation of wave packet by a water drop is worthy of further study.

3.4 The repeatability of experiment

A similar process to that in Video 1 is recorded in Video 2, from which we got C_{ge}=15.4 cm/s, C_{pe}=25.2 cm/s.

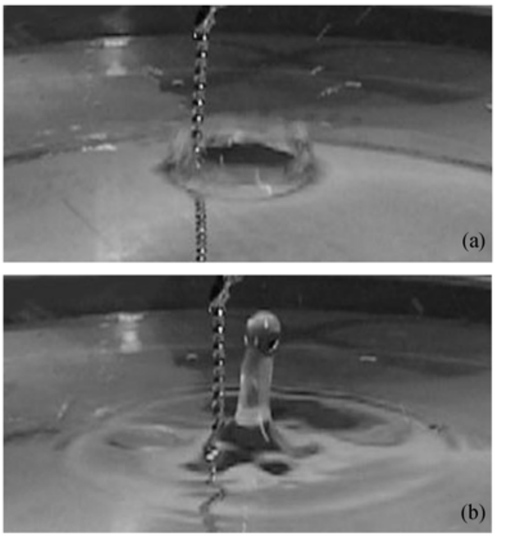

Figure 6 The shapes splashed by a water drop. (a) A crown, 90 ms after the fall of the drop; (b) a tower-like water column with a drop on the spire after 150 ms. A string of beads above the water is used as a reference and the focusing aim of camera. The diameter of the bead is 2.3 mm.

4 Conclusions

(i) The main reason why the packet can propagate with low diffusion is that the wavelength of the carrier waves is very close to the minimum point of the dispersion curve of group velocity.

(ii) Figure 4 shows the propagation of the carrier relative to the envelop intuitively, which can be use as an example to explain the group velocity and the phase in the textbook of general physics.

(iii) It is worthy of further study on the conditions and the process of forming this kind of wave packet.

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1 Le Mehaute B. Gravity-capillary rings generated by water drops. J Fluid Mech, 1988, 197: 415—427
2 Crapper G D. Introduction to Water Waves. Chichester, West Sussex, England: E. Horwood; New York: Distributor, Halsted Press. 1984
3 Craeye C, Sobieski P W, Bliven L F, et al. Ring-waves generated by water drops impacting on water surfaces at rest. IEEE J Oceanic Eng, 1999, 24(3): 323—332
4 Longguet-Higgins M S. Capillary-gravity waves of solitary type and envelope solitons on deep water. J Fluid Mech, 1993, 252: 703—711
5 Longguet-Higgins M S, Zhang X. Experiments on capillary-gravity waves of solitary type on deep water. Phys Fluids, 1997, 9: 1963—1968
6 Zhu G Z. Research on water waves. Academic report in State Key Laboratory of Acoustics. July 6, 2006
7 Russell S J. Report on Waves. In: 14th Meeting of the British Association for Advancement of Science. London: John Murray, 1844. 311—390
8 Rayleigh L. The Theory of Sound. 2nd ed. New York: Dover, 1945
9 Rein M. Phenomena of liquid drop impact on solid and liquid surfaces. Fluid Dynam Res, 1993, 12: 61—93
10 Cossali G E, Marengo M, Coghe A, et al. The role of time in single drop splash on thin film. Exp Fluids, 2004, 36: 888—900
11 Honey E M, Kavehpour H P. Astonishing life of a coalescing drop on a free surface. Phys Rev E, 2006, 73: 027301